Eco 362: Economic Growth
Fall 2013
Solutions for Problem Set 2

Note: all Weil question numbers refer to the 3rd edition textbook.

Question 1: Weil Chapter 4, Q4

Solution: In a randomized controlled trial, one would have to randomly vary either the quantity or the quality of children in a treatment group and compare the children in this treatment group to children in a control group. For example, providing enhanced education to the treatment group represents an exogenous downward shock to the cost of having higher quality children. Providing family planning to a treatment group would represent an exogenous downward shock to the quantity of children. Using twins would be a good natural experiment. Since twin births are basically random, they provide an identifying exogenous variation of quantity. One can compare the quality of children who were born as twins to the quality of children who were born alone.

Question 2 Consider the Malthus model where the production function is given by $Y_t = AT^{\beta}L_t^{1-\beta}, 0 < \beta < 1$

a) Derive the equation for the evolution of per capita income and population

b) Derive the steady state of per capita income and population

b) Discuss the effects on the economy of a **one time, permanent** increase in the **amount of land** (T). Discuss the effects graphically and in words.

Answer:

Assumptions:

- The production function is given by $Y_t = AT^{\beta}L_t^{1-\beta}, 0 < \beta < 1$
- T (land) is constant
- $L_{t+1} = (\pi y_t)\, L_t$ $\pi > 0$ i.e. Population growth rate depends positively on people's income. The higher the income the higher the population growth rate.

a) Consider the evolution of per capita income

$$
\begin{aligned}
y_{t+1} &= \frac{Y_{t+1}}{L_{t+1}} = \frac{AT^{\beta}L_{t+1}^{1-\beta}}{L_{t+1}} \\
y_{t+1} &= A\left(\frac{T}{L_{t+1}}\right)^{\beta} \\
y_{t+1} &= A\left(\frac{T}{\pi y_t L_t}\right)^{\beta} \\
y_{t+1} &= \frac{1}{\pi^{\beta}}\frac{1}{y_t^{\beta}}A\left(\frac{T}{L_t}\right)^{\beta} \\
y_{t+1} &= \frac{1}{\pi^{\beta}}\frac{1}{y_t^{\beta}}y_t \\
y_{t+1} &= \frac{1}{\pi^{\beta}}y_t^{1-\beta}
\end{aligned}
$$

Consider the evolution of population

$$
\begin{aligned}
L_{t+1} &= (\pi y_t)\, L_t \\
L_{t+1} &= \pi A\left(\frac{T}{L_t}\right)^{\beta} L_t \\
L_{t+1} &= \pi AT^{\beta}L_t^{1-\beta}
\end{aligned}
$$

b) At the steady state, $y_t = y_{t+1} = y^{SS}$

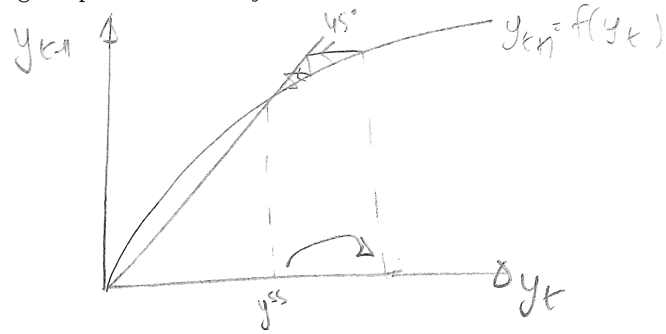
$$
\begin{aligned}
y^{SS} &= \frac{1}{\pi^{\beta}}\left(y^{SS}\right)^{1-\beta} \\
\frac{y^{SS}}{\left(y^{SS}\right)^{1-\beta}} &= \frac{1}{\pi^{\beta}} \\
y^{SS} &= \frac{1}{\pi}
\end{aligned}
$$

At the steady state $L_t = L_{t+1} = L^{SS}$

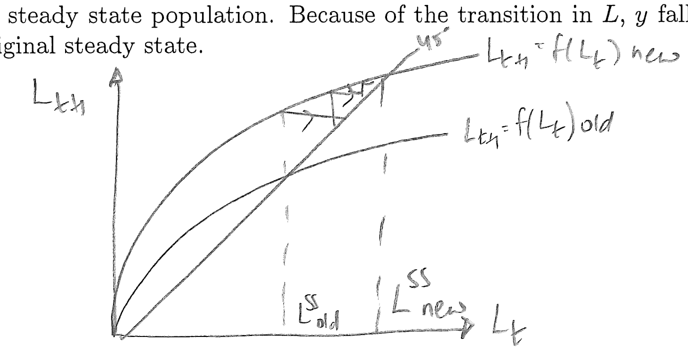
$$
\begin{aligned}
L^{SS} &= \pi AT^{\beta}\left(L^{SS}\right)^{1-\beta} \\
\frac{L^{SS}}{\left(L^{SS}\right)^{1-\beta}} &= \pi AT^{\beta} \\
L^{SS} &= (\pi A)^{\frac{1}{\beta}}\, T
\end{aligned}
$$

c) There is a **one time, permanent** increase in the **amount of land, i.e.** $T_{old} < T_{new}$. Assume we start out in the steady state.

Effects on y : When the amount of land increases today, for the same population (L_t) because each worker has more land to work with, the marginal product of L increases. This causes an increase in the per capita income today (notice $y_t = A\left(\frac{T}{L_t}\right)^{\beta}$). This increases moves us along the $y_{t+1} = f(y_t)$ curve and takes us above steady state. We get a jump up in y to a higher level. At the higher y, people have more children and this gradually because of diminishing marginal product causes y to transition down to the same old steady state



Effects on L : When the amount of land increases today, for the same population (L_t) per capita income today rises. Because of this people can afford to have more children. Thus for any L_t we now get a higher amount of y_t than before, making L_{t+1} higher at every point causing the $L_{t+1} = f(L_t)$ curve to shift outwards. People have more kids and this decreases y. However the larger amount of T can sustain a bigger population at every L_t so people continue to have more kids. This raises the population gradually till we reach the new higher steady state population. Because of the transition in L, y falls back to the original steady state.



The short run effect is a jump upwards in y and a gradually decrease back towards the old y^{SS}. There is no long run change in y. In the short run, L gradually increases. The long run effect is a positive level change in L^{SS}.